



- Children extend previous years' work by adding and subtracting numbers with up to four digits, using mental and written methods, including columnar addition and subtraction. They keep practising mental methods of addition and subtraction as well as written methods, performing calculations increasingly quickly and confidently. They continue using estimation as well as inverse operations to help check answers.
- Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- Estimate and use inverse operations to check answers to a calculation
- Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

ADDITION

Reordering Children should understand why it is more efficient when adding to reorder numbers and to put the largest number first.

Knowledge of number bonds to support bridging

$$48 + 7 = 48 + 2 + 5 = 50 + 5 = 55$$

$$58 + 47 \text{ becomes } 58 + 2 + 45 = 105$$

$$2.8 + 1.7 \text{ becomes } 2.8 + 0.2 + 1.5 = 4.5$$

Partition both numbers then add and recombine.

$$\begin{aligned} 145 + 123 &= 100 + 100 + 40 + 20 + 5 + 3 \\ &= 200 + 60 + 8 \\ &= 268 \end{aligned}$$

Partitioning just the smaller addend

$$234 + 122 = 234 + 100 + 20 + 2$$

Doubles and near doubles

$$143 + 145 = \text{double } 140 + 8 = 288$$

$$6.2 + 6.3 = \text{double } 6 + 0.5 = 12.5$$

Compensating - adding a close multiple of 10 (e.g. 18, 19, 21, 22)

$$156 + 18 \text{ becomes } 156 + 20 - 2 = 154$$

$$2 \frac{1}{2} + 1 \frac{3}{4} \text{ becomes } 2 \frac{1}{2} + 2 - \frac{1}{4}$$

Deriving new facts from known

Decimal add based on $3 + 7 = 10$

Quick recall of number bonds to 100 and be able to apply to other calculations including decimals, e.g. $4.3 + ? = 5$

SUBTRACTION

Bridging through ten and multiples of ten should also be used when subtracting.

$$43 - 17 \text{ becomes } 43 - 10 - 3 - 4 = 26$$

Counting on to find the difference.

Count on because the minuend and subtrahend are close together

$$3641 - 2991 =$$

$$5001 - 1997 =$$

$$0.63 - 0.48 =$$

Partition before count back if minuend and subtrahend are further apart ~

$$464 - 129 \text{ becomes } 464 - 100 = 364 \quad 364 - 20 = 344 \quad 344 - 4 = 340 \quad 340 - 5 = 335$$

Compensating ~ subtracting a close multiple of 10 (e.g. 18, 19, 21, 22)

$$\begin{aligned} 72 - 19 &= 72 - 20 + 1 \\ &= 52 + 1 = 53 \end{aligned}$$

$$97 - 58 \text{ becomes } 97 - 60 + 2 =$$



Addition - Add up to 4 digits, using formal written methods

When teaching the stages in progression, start with models and make connections with the expanded and formal methods at the same time. For example using counters and/or Dienes alongside the expanded and formal methods. Children should be able to explain what is happening when they carry. Place emphasis on the ability to explain and reason about the mathematics behind the method. E.g. Ask 'What's the same and what's different?'

MODELS

REVIEW EXPANDED

$200 + 30 + 4$	234
$500 + 20 + 7$	527
$\hline 700 + 60 + 1$	$\hline 761$
10	1

→

FORMAL

$\begin{array}{r} 367 \\ + 85 \\ \hline 452 \\ \hline 11 \end{array}$	<p>Extend to decimal - link to money.</p> <p>£14.75 + £2.40 14.73</p> <p>14.75</p> <p><u>+2.20</u></p> <p><u>16.95</u></p> <p style="margin-top: 10px;">Encourage use of ESTIMATION throughout.</p>
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Subtraction - Up to 4 digits using a formal written method

Children should also be encouraged to look at the difference between two numbers before making a decision about how to use the number line e.g. small or large difference - count on or count back.

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SUBTRACTION

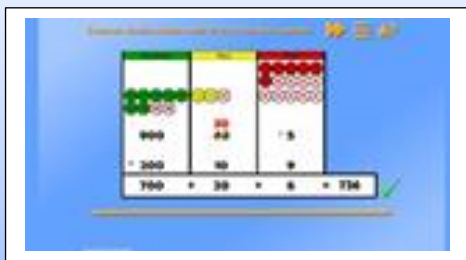
MODELS

BAR MODEL AND
NUMBER LINES

$$100 - 70 = 30$$

$$82 - 49 = 33$$

$$512 - 386 = (4 + 10 + 100 + 12) = 126$$

$$512 - 386 = 126$$


Horizontal expansion:

e.g. $458 - 124$

$$\begin{array}{r} 400 \ 50 \ 8 \\ - 100 \ 20 \ 4 \\ \hline 300 + 30 + 4 = 334 \end{array}$$

Horizontal expansion with adjustment: e.g. $754 - 286$

$$\begin{array}{r} 600 \ 140 \ 14 \\ - 200 + 50 + 4 \\ \hline 400 + 60 + 8 = 468 \end{array}$$

LEADING TO FORMAL WRITTEN

$\begin{array}{r} 8 \ 7 \ 4 \\ - 5 \ 2 \ 3 \\ \hline 3 \ 5 \ 1 \end{array}$ <p>Answer: 351</p>	$\begin{array}{r} 8 \ 12 \ 1 \\ \cancel{9} \ \cancel{3} \ 2 \\ - 4 \ 5 \ 7 \\ \hline 4 \ 7 \ 5 \end{array}$ <p>Answer: 475</p>
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- Developing mental fluency in multiplication and division.

Multiplication facts (including squares) to 12×12 .

Factor pairs

E.g. factor pairs of 12 = 1, 12, 2,6,3,4,
therefore 1×12 , 2×6 , $3 \times 4 = 12$

Multiples of 2, 3, 4,5,6,8 10, 25 and 50

Divisibility/multiple rules

E.g. multiple of 3 if '*addition of digit roots sum to multiple of 3*'

$48 = 4 + 8 = 12$ therefore 48 multiple of 3.

Multiple of 6 is even and digit roots sum to multiple of 3
e.g. $72 = \text{even} = 7 + 2 = 9$ therefore 72 multiple of 6.

Recall prime numbers to 10.

Use multiplication facts to identify if a number from 1 to 100 is a prime number.

Recognise multiplication as **repeated addition** -E.g. plant growth is 3cm per day therefore over 7 even days plant will be 21cm

Use above for approximation - operating first with all the digits that have the greatest value (left to right)

E.g. $42 \times 4 \approx 160$

$123 \times 4 \approx 480$

Children should have implicit understanding of commutative law

E.g. $a = 1$, $b = 2$, $c = 3$

$a \times b = b \times a$ i.e. $1 \times 2 = 2 \times 1$

Division facts (+ square roots) derived from multiplication tables to 12×12

Factor pairs

E.g. factor pairs of 12 = 1, 12, 2,6,3,4,
therefore 12 is divisible by 1, 12, 2, 6,3,14,

Multiples of 2, 3, 4,5,6,8 10, 25 and 50

Divisibility/multiple rules

Divisibility rules:

E.g. *an even dividend divided by 2 will give a quotient which is even.*

An even whole number with a divisor of 6 will give a whole number quotient if dividend is an even number and the digit roots sum to a multiple of 3.

$486 \div 6$ 486 even; $4 + 8 + 6 = 18$ multiple of 3
therefore divisible by 6.

Use prime numbers to decide if a number up to 100 is divisible by more than 0, 1, and itself.

Recognise division as **repeated subtraction** but not always **discrete**. E.g. Plant growth over one week is 21 cm therefore daily growth is approx. 3cm.

Use above for approximation - operating first with all the digits that have the greatest value (left to right)

E.g. $168 \div 4 \approx 40$

$469 \div 4 \approx 110$

Children should know that commutative law are **not applicable** for division

Place value:

Use place value knowledge to multiply by 10,100 or 1000.e.g.

$$10 \times 18 = 180$$

Use place value to derive unknown facts e.g. $3 \times 40 = 120$

therefore $3 \times 400 = 3 \times 40 \times 10 = 1200$, $30 \times 40 = 3 \times 4 \times 100$.

Doubling and halving:

Use doubling/halving to calculate unknown facts

E.g. to multiply by 4 - double and double again.

To multiply by 5, multiply by 10 and halve.

$$60 \times 4 = 120 \text{ therefore } 30 \times 4 = 60$$

Doubling and halving balancing

E.g. $24 \times 6 = (\text{halve one side/ double other side})$

$$12 \times 12$$

Using related facts

E.g. $19 \times 4 = 20 \times 4 - 4 = 76$

Partitioning: Use partitioning to find partial products then sum

E.g. $23 \times 6 =$

$$20 \quad 3$$

$$20 \times 6 \quad + \quad 3 \times 6$$

$$120 \quad + \quad 18 = 138$$

Place value

Use place value to divide by 10,100 or 100 e.g. $36 \div 9 = 4$

therefore $360 \div 90 = 4$; $360 \div 9 = 40$

Use place value to derive unknown facts e.g. $60 \div 5 = 12$

therefore $600 \div 5 = 120$

Doubling and halving:

Use doubling and halving to calculate unknown facts.

E.g. to divide by 4 - halve and halve again.

$$280 \div 4 = 280 \div 2 \div 2 = 70$$

Partitioning:

Use partitioning to find partial quotients then sum

e.g. $138 \div 6 =$

$$(120 \div 6) + (18 \div 6)$$

$$20 \quad + \quad 3 = 23$$

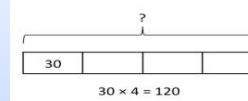


Multiplication - TU by U, HTU by U, ThHTU by U

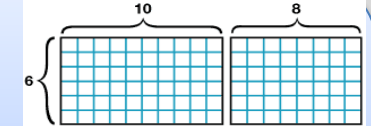
Children should not be using column method until secure with concept. Grid method is an excellent stepping stone to column method and should be taught. Same /different approach should be applied. Children's' understanding that multiplication is associative should be highlighted here.

MODELS AND REPRESENTATIONS

Partition/decompose 2-digit numbers;
Model area representations using distributive property and partial product arrays. Use concrete resources then children represent arrays using pencil and paper - then apply to grid method. Emphasis on partial products as this aids transition to conceptual understanding of formal written method.



Counters and Large PV



$$18 \times 6 = (10 \times 6) + (8 \times 6)$$

$$= 60 + 48$$

$$= 108$$

Models - use arrays

MULTIPLICATION

MODELS TO WRITTEN METHOD

INFORMAL WRITTEN TO WRITTEN

x	30	5
7	210	35

$$210 + 35 = 245$$

Grid:

$$123 \times 5$$

$$\begin{array}{r|l} \times & 100 & 20 & 3 \\ 5 & 500 & 100 & 15 \end{array}$$

Expanded Written

1	2	3		
x			5	
	1	5		(3 x 5)
	1	0	0	(20 x 5)
	5	0	0	(100 x 5)
	6	1	5	

Contracted Written

1	2	3		
x			5	
	6	1	5	
	1	1		

Same/different questioning used for comparison and making links.

FORMAL WRITTEN

24 x 6 becomes

$$\begin{array}{r} 24 \\ \times 6 \\ \hline 144 \\ 2 \\ \hline \end{array}$$

Answer: 144

342 x 7 becomes

$$\begin{array}{r} 342 \\ \times 7 \\ \hline 2394 \\ 2 \\ \hline \end{array}$$

Answer: 2394

2741 x 6 becomes

$$\begin{array}{r} 2741 \\ \times 6 \\ \hline 16446 \\ 4 \\ \hline \end{array}$$

Answer: 16446



Division - $TO \div O$ and $HTO \div U$

$TO \div O$ and $HTO \div U$ with no remainder or remainder expressed as remainder e.g. $13r2$. Children should be progressing to using a formal written method for short division with $TO \div O$ and $HTO \div U$ with no remainder or remainder expressed as remainder e.g. $13r2$. However - huge emphasis should still be placed on using numbers lines and using resources such as place value counters and arrays to support conceptual understanding of formal methods. Children should also be taught what to do with remainder in context of worded problem.

DIVISION

MODELS TO WRITTEN METHOD

MODELS AND REPRESENTATIONS

NUMBER LINE AND INFORMAL WRITTEN

Use same different questions here to compare approaches. Ensure pupils understand the relationship between multiplication and division.

FORMAL WRITTEN METHOD SHORT DIVISION

$492 \div 4$

When working with children - key questions are: How many groups of ____ are there? ____ group of ____ in answer box.

