

# **Progression towards a Standard Written method of Calculation**

The National Numeracy Strategy provides a structured and systematic approach to teaching number. There is a considerable emphasis on teaching mental calculation strategies. Up to the age of 9 (Year 4) informal written recording should take place regularly and is an important part of learning and understanding. More formal written methods should follow only when the child is able to use a wide range of mental calculation strategies.

## **REASONS FOR USING WRITTEN METHODS**

- To aid mental calculation by writing down some of the numbers and answers involved
- To make clear a mental procedure for the pupil
- To help communicate methods and solutions
- To provide a record of work to be done
- To aid calculation when the problem is too difficult to be done mentally
- To develop and refine a set of rules for calculations

## **WHEN ARE CHILDREN READY FOR WRITTEN CALCULATIONS?**

### **Addition and subtraction**

- Do they know addition and subtraction facts to 20?
- Do they understand place value and can they partition numbers?
- Can they add three single digit numbers mentally?
- Can they add and subtract any pair of two digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?

### **Multiplication and division**

- Do they know the 2, 3, 4, 5 and 10 time table
- Do they know the result of multiplying by 0 and 1?
- Do they understand 0 as a place holder?
- Can they multiply two and three digit numbers by 10 and 100?
- Can they double and halve two digit numbers mentally?
- Can they use multiplication facts they know to derive mentally other multiplication facts that they do not know?
- Can they explain their mental strategies orally and record them using informal jottings?
- Can they partition by multiples of the divisor and by place value

If children cannot access age appropriate objectives, track back to previous year groups as necessary. If children are working beyond expectations then move forwards to the next year group.

It is essential that children's mental methods in all four operations are very secure and they are able to use a variety of strategies as appropriate.

## Using the Policy

Each area of calculation is in a different colour:

**Addition:** e.g. pink

**Subtraction:** e.g. yellow

**Multiplication:** e.g. purple

**Division:** e.g. green

Each Year is on a different page and the layout is the same for all year groups.

### **Related objectives:**

- Facts
- Place Value
- Understanding

These are the areas that are closely linked to understanding calculation for any particular year group. These objectives need to be taught as the main part of the lesson but should also be part of an on-going programme of mental and oral starters that support the teaching and learning of each of the four operations

### **Differentiation**

**Progression in calculations:** The progression details the calculations appropriate for that year group, with some room for extension. Children's fluency in calculation should be given greater emphasis than their ability to use an informal or formal written method. This is particularly important for Year 4 and Year 5 teachers who may be tempted to move the children on to an informal method before they are fluent in a range of strategies in mental calculation.

**Strategies:** The strategies that children should be aware of are detailed for each year group. Children should be encouraged to use a range of strategies and to consider the most appropriate strategy for any given calculation. Children's ability to consider a range of strategies should be given greater emphasis than their ability to use either an informal or formal written method. This is particularly important for Year 4 and Year 5 teachers who may be tempted to move the children on to an informal method before they are fluent in a range of strategies in mental calculation.

**Models, images and resources:** These facilitate access to strategies. They are a visual and concrete image to support teachers' explanations for any given strategy e.g. jottings and empty number lines.

Children can then use these models to support their thinking and aid calculation. Children should be encouraged to become as efficient as possible in their jottings until they no longer need to record to support their thinking. They then move on to more challenging calculations where they do need to use jottings to aid calculation.

E.g.  $16+7$

Child A – uses fingers to count on 7 from 16;

Child B – uses a number track to demonstrate jumping 4 from 16 to 20 and then on 3 to 23;

Child C – uses an empty number line to jump 4 from 16 to 20 and then on 3 to 23;

Child D – can calculate by splitting 7 into 4 and 3, quickly, without using jottings. They will then move onto calculating e.g. 26 and 47.

It is therefore important that teachers are aware of children's strategies and their fluency when diagnosing their level of competency in calculation. Child A and Child D may appear to be working at the same level because they can do the calculation correctly.

# Addition

## Stage 1

### Related objectives: Facts, Place Value and Understanding (Mental/ Oral starters)

- Within the range of 0 – 30 say the number that is 1 or 10 more or less than any given number
- Know by heart all pairs of numbers with a total of 10
- To recognise + and = signs in simple number sentences
- To partition at least a teens number
- To understand addition can be done in any order
- Understand and use related **vocabulary**: more, add, sum, total, altogether, equals, sign

### Progression in calculations

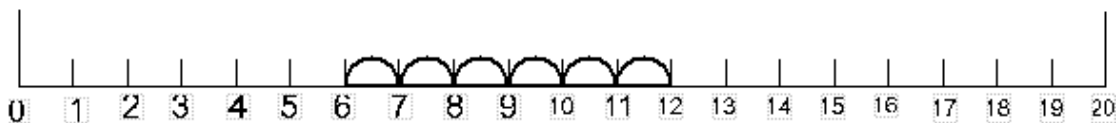
- U + U
- teen numbers + U
- 2 digit + U
- 2 digit + U crossing the tens boundary
- teen numbers + teen numbers

### Strategies

- Combining two sets counting all
- Put larger number first and find total by counting on
- Use partitioning for a teen number + teen number

### Models and Images

- Using counters, blocks, fingers and bead string
- Use laminated number lines and hundred squares to aid calculation by drawing jumps to show the addition
- $6 + 6 =$



## Stage 2

### Related objectives (Mental/ Oral starters)

#### Facts and Place Value

- Know what a 2 digit number represents, including 0 as a place holder, and partition a 2 digit number into a multiple of 10 and 1s
- Say the number that is 1 or 10 more or less than any given 2 digit number
- Know all addition and subtraction facts for each number to 10

#### Understanding

- Subtraction is the inverse of addition ( $5 + \_ = 21$   $21-5=16$ )
- Know that addition can be done in any order
- Understand and record using + and = sign
- Understand and use related **vocabulary**: more, add, sum, total, altogether, equals, sign

### Progression in calculations

- $16+3$  (Teen and unit (not crossing boundary))
- $16+7$  (e.g. by counting on in ones) (crossing boundary)
- $16+7$  (e.g. by splitting 7 (requires knowledge of number bonds into 4 and 3))
  
- $14+11$  (Teen and teen not crossing boundary – near multiple of 10)
- $15+16$  (Teen and teen crossing boundary)
  
- $20+12$  (Tens and teen)
- $25+19$  (TU and teen crossing boundary- near multiple of 10)
  
- $20+20$  (Tens and tens)

### Strategies

- Put larger number first and count on
- Partition, add and recombine
- Count on in tens and ones  $43 + 32 = 43 + 10 + 10 + 10 + 1 + 1 = 75$
- Use knowledge of bonds to 10 ( $24 + 8 = 24 + 6 + 2 = 30 + 2 = 32$ )
- Adding near multiple of 10
- Doubles and near doubles (e.g.  $6+7$ ,  $40 + 39$ )

### Models and Images (Use any of the models and images below to support the teaching of the strategies above)

1. Use of bead string and number track
2. Use of hundred square to show jumps of T, U, TU
3. Simple jottings:  
 $16+13 = 10 + 10 + 6 + 3 = 29$
4. Blank number lines:  
 $26 + 23 =$



## Stage 3

### Related objectives (Mental/ Oral starters)

#### Facts and Place Value

- Partition 3 digit numbers into multiple of 100, 10 and 1
- Say the number that is 1, 10 or 100 more or less than any given 2 or 3 digit number
- Know all addition and subtraction facts for each number to 20

#### Understanding

- Subtraction is the inverse of addition ( $12 + \_ = 36$ ,  $36 - 12 = 24$ )
- Know that addition can be done in any order
- Understand and record using + and = sign
- Understand and use related **vocabulary**: more, add, sum, total, altogether, equals, sign

### Progression in calculations

$20+20$

$20+12$

$14+11$

$16+7$  (splitting 7 into 4 and 3)

$15+16$

$25+19$

$67+24$

$70+50$  (crossing 100s boundary)

$80+56$

$86+57$

$500+300$

$345+ 300$

$356+427$

### Strategies

**Encourage mental fluency: children should not be over reliant on the hundred square and should partition mentally or visualise the empty number line in their heads for TU+TU/U. For children who still need jottings at this stage, encourage efficient strategies e.g bigger jumps\***

- Count on in tens & ones e.g.  $43 + 32 = 43 + 10 + 10 + 10 + 1 + 1 = 75$   
progressing to  $43+30+2^*$
- Use knowledge of bonds to 10 ( $24 + 8 = 24 + 6 + 2 = 30 + 2 = 32$ )
- Adding near multiple of 10
- Doubles and near doubles e.g.  $36+35$ ,  $60+70$ ,  $18+16$

### Models and Images

TU + TU, developing to HTU + TU or HTU + HTU. Use hundred squares, blank number lines and simple partitioning and recombining (as year 2, with more challenge):

1. By counting on in multiples of 10 or 1:  $86 + 57 = 86 + 50 + 7 = 136 + 7 = 143$
2. And multiples of 100, 10 or 1:  $356+427= 427+ 300 + 50 + 6 = 727 + 50 + 6 = 777 + 6 = 783$
3. Adding a near multiple of 10:  $35+19=35+20-1$

35

54

55

## Stage 4

### Related objectives (Mental/ Oral starter)

#### Facts and Place Value

- Partition 4 digit numbers into multiple of 1000, 100, 10 and 1
- Say the number that is 1, 10,100 or 1000 more or less than any given 2, 3 or 4 digit number
- Derive quickly all pairs of numbers that total 100 eg. 60 +40, 75+25, 38+62

#### Understanding

- Subtraction is the inverse of addition ( $33+_ = 75$ ,  $75-33=42$ )
- Knowledge that addition can be done in any order
- Understand and record using + and = sign
- Understand and use related **vocabulary**: more, add, sum, total, altogether, equals, sign, **increase**, **inverse**

### Progression in calculations

$20+20$

$20+12$

$14+11$

$16+7$  (splitting 7)

$15+16$

$25+19$

$67+24$

$70+50$  (crossing 100s boundary)

$80+56$

$86+57$

$500+300$

$345+ 300$

$356+427$

$3200+600$

$3200+900$

### Strategies (Use models and images as for Year 3)

- Put larger number first
- Count on in hundreds, tens & ones e.g.  $225 + 325 = 325 + 200 + 20 + 5$
- Use knowledge of bonds to 10 ( $24 + 8 = \mathbf{24} + \mathbf{6} + 2 = \mathbf{30} + 2 = 32$ )
- Adding near multiple of 10
- Doubles and near doubles (e.g.  $38+36$ ,  $160+170$ ,  $380+380$ )

### Written Method (if children are struggling to learn this procedure quickly go back to models and images for stage 3)

Introduce informal methods for addition. TU + TU to learn method, adding least significant digit first. As soon as children are secure with this method, extend to HTU + TU, still working in the expanded form.

$47+76$

$40+7$

$70+6$

---

 $13$

$110$

---

 $123$

$347+276$  (to model procedure)

$300+40+7$

$200+70+6$

---

 $13$

$110$

---

 $500$

---

 $623$

Leading quickly to:

$368$

$+ 493$

---

 $11$

$150$

---

 $700$

---

 $861$

## Stage 5

### Related Objectives: Facts, Place Value and Understanding (Mental /Oral Starter)

- Partition 4 digit numbers into multiple of 1000, 100, 10 and 1
- Say the number that is 1, 10, 100 or 1000 more or less than any given 2, 3 or 4 digit number
- Count forward and backwards in steps of 0.1, 0.2, 0.3 ...
- Derive quickly decimals that total 1 e.g.  $0.2+0.8$ ,  $0.75+0.25$ ,  $0.32+0.68$
- Derive quickly pairs that total 100
- Add several numbers e.g. single digits, or multiples of ten e.g.  $40 + 50 + 80$
- Develop further relationship between addition and subtraction
- Understand and use related **vocabulary**: more, add, sum, total, altogether, equals, sign, **increase, inverse**

### Progression in calculations

20+20	500+300
20+12	345+ 300
14+11	356+427
16+7 (splitting 7)	3200+600
15+16	3200+900
25+19	<b>Decimals</b>
67+24	5.4 + 2.1
70+50 (crossing 100s boundary)	6.4 +3.8
80+56	0.63+0.28
86+57	

### Strategies (Use models and images as for Year 3)

- Add nearest multiple of 10 or 100 and adjust
- Count on (use empty number line for children finding decimals difficult)
- Partition and use knowledge of number bonds
- Doubles and near doubles

### Written method

Expanded method for HTU+HTU, leading to ThHTU + HTU and decimal numbers.

368	2346
+ 493	+ 3187
-----	-----
11	13
150	120
700	400
-----	-----
861	5000
-----	-----
	5533
	-----

If children are secure with this, use compact (carrying) method (NB School must be consistent in method of recording for carrying.):

368
+ 493
-----
861
-----
11
-----

## Stage 6

### Related Objectives: Facts, Place Value and Understanding (Mental/ Order starter)

- Partition 4 digit numbers into multiple of 1000, 100, 10 and 1
- Say the number that is 1, 10, 100 or 1000 more or less than any given 2, 3 or 4 digit number
- Count forward and backwards in steps of 0.1, 0.2, 0.3 ... and 0.25
- Derive quickly decimals that total 1 eg.  $0.2+0.8$      $0.75+0.25$      $0.32+0.68$
- Derive quickly pairs that total 100
- Add nearest multiple of 10 or 100 and adjust
- Develop further relationship between addition and subtraction
- Understand and use related **vocabulary**: more, add, sum, total, altogether, equals, sign, **increase**, **inverse**

### Progression in calculations

$20+20$

$20+12$

$14+11$

$16+7$  (splitting 7)

$15+16$  (crossing 10s boundary)

$25+19$

$67+24$

$70+50$  (crossing 100s boundary)

$80+56$

$86+57$

Add several numbers e.g. single digits or multiples of ten e.g.  $40 + 50 + 80$

$500+300$

$345+ 300$

$356+427$

$3200+600$

$3200+900$

#### **Decimals**

$5.4 + 2.1$

$6.4 +3.8$

$0.63+0.28$

Add several numbers with different number of digits eg  $23 + 139 + 4 + 205$

### Strategies (Use Models and Images as for Year 3)

- Add nearest multiple of 10 or 100 and adjust
- Count on partitioning the smaller number e.g.  $154+775=775+100+50+4$
- Doubles and near doubles
- Knowledge of number bonds

### Written Method

Use of expanded method for ThHTU + ThHTU, and if children are secure, using compact method (carrying), including decimals

$$\begin{array}{r} 2686 \\ +1385 \\ \hline 4071 \\ \hline 11 \end{array}$$



# Subtraction

## Stage 1

### Related objectives

#### Facts and Place Value

- Within the range of 0 – 30 say the number that is 1 or 10 more or less than any given number
- Know by heart all pairs of numbers with a total of 10
- To partition at least a teens number

#### Understanding

- To recognise and record – and = signs in simple number sentences
- Use knowledge that subtraction is inverse of addition
- Understand & use related **vocabulary**: take away, subtract, how many are left, how much less is..than..., difference between, how much more is..than..., how many more to make
- Understand that e.g.  $10-3 = 7$ ,  $20 - 3 = 17$ ,  $30-3=27$

### Progression in calculations

- $7 - 1$
- $7 - 3$
- $15 - 3$
- $10 - 3$
- $20 - 6$
- $15 - 10$
- $15-7$

### Strategies

- Count how many are left after some have been taken away
- Count back on a number line
- Count on to see how many were removed or how many will make a given number

### Models and Images

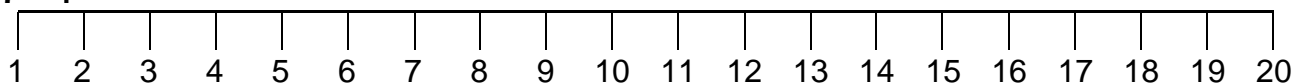
- Counters, block and bead string
- Use of hundred square and number track/number line

Children need experience of counting on and back, e.g.  $7 - 4$ , by counting back from 7 and then by counting on from 4 to 7.

**NB** Counting on is a progression from counting back (when counting on use vocabulary of *how many more? And difference between*)

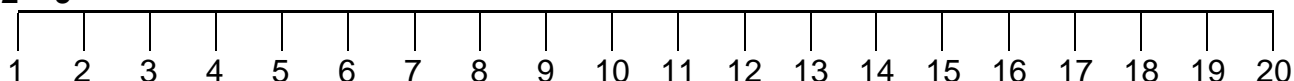
### Counting back

$$17 - 4$$



### Counting on

$$12 - 9$$



## Stage 2

### Related objectives (Mental / Oral Starter)

#### Facts and Place Value

- Use known number facts to add and subtract mentally (using jotting where appropriate)
- Know what a 2 digit represents including 0 as a place holder and partition a 2 digit number into a multiple of 10 and 1s;
- Say the number that is 1 or 10 more or less than any given 2 digit number;
- Know all addition and subtraction facts for each number to 10
- Recognise when it is easier to count up or back when finding the difference

#### Understanding

- Use knowledge that subtraction is inverse of addition ( $\_ - 4 = 6$   $6 + 4 = 10$ )
- Record mental subtraction using  $-$  and  $=$  signs
- Use related vocabulary
- Understand & use related **vocabulary**: take away, subtract, how many are left, how much less is..than.., difference between, how much more is..than.., how many more

#### Progression in calculations

- $20 - 6$
- $15 - 10$
- $15 - 7$  (splitting 7 into 5 and 2)
- $40 - 20$
- $45 - 20$
- $45 - 23$
- $40 - 19$
- $45 - 27$

#### Strategies

- Count on or back on empty number lines
- Partitioning
- Bridging through 10 or a multiple of 10 (using knowledge of number bonds for numbers 1-10)

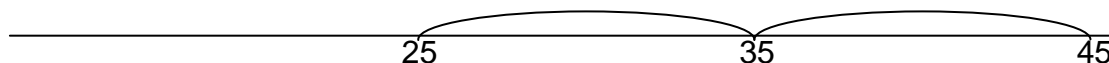
#### Models and Images

As year 1, children need experience of counting on and back, using larger numbers, but should be able to decide which strategy is most appropriate, e.g.  $23 - 17$ : by counting on from 17 to 23,  $23 - 6$ : by counting back 6 from 23.

Children should be using number tracks, hundred squares and empty number lines to calculate.

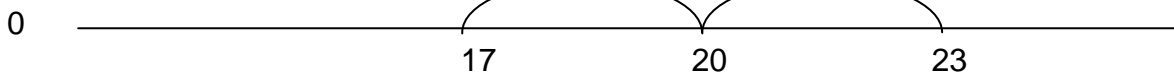
#### **Counting Back**

45-20



#### **Counting on**

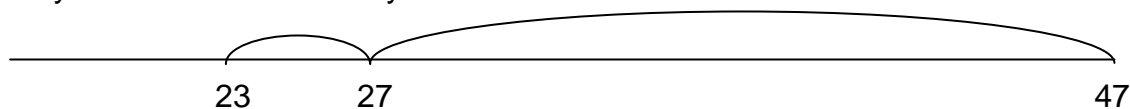
23 - 17



Children working with bigger numbers may use partitioning method:

e.g.  $47 - 24 =$                        $47 - 20 = 27$                        $27 - 4 = 23$

This may be carried out mentally or blank number line:



## Stage 3

### Related objectives ( Mental / Oral Starter)

- Partition 3 digit numbers into multiple of 100, 10 and 1
- Say the number that is 1, 10 or 100 more or less than any given 2 or 3 digit number
- Know all addition and subtraction facts for each number to 20
- Number bonds to 100

### Understanding

- Use knowledge that subtraction is inverse of addition ( $\_ - 10 = 36$ ,  $36 + 10 = 46$ )
- Record mental subtraction using  $-$  and  $=$  signs
- Understand & use related **vocabulary**: take away, subtract, how many are left, how much less is..than.., difference between, how much more is..than.., how many more

### Progression in calculations

45 – 23	819 – 200
15 – 7	1200 – 500
40 – 19	900 – 7
45 – 27	4000 – 3
120 – 30	905 – 7
146 – 50	4641 – 3
800 – 500	<b>372 – 368</b> (small difference)

### Strategies

- Counting on and back on empty number line in ones, tens and hundreds
- Partitioning

### Models, Images and Resources

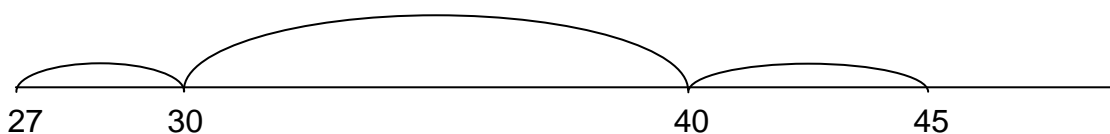
100 square, number lines and bead string

TU – TU, developing to HTU – TU, and HTU - HTU

It is important that children should decide which method of subtraction is appropriate for each calculation, and use the blank number line approach (see overleaf)

counting on

45-27

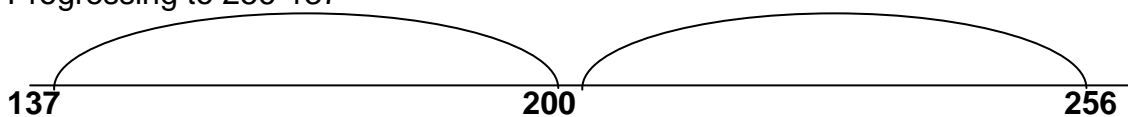


Counting Back

146 - 18



Progressing to 256-137



**Children should not be introduced to the vertical/column method of subtraction in Year 3.**

## Stage 4

### Related objectives: Facts, Place Value and Understanding (Mental / Oral Starter)

- Use known number facts to add and subtract mentally (using jotting where appropriate)
- Partition 4 digit numbers into multiple of 1000, 100, 10 and 1
- Say the number that is 1, 10, 100 or 1000 more or less than any given 2, 3 or 4 digit number
- Derive quickly all pairs of numbers that total 100 eg, 60 40 75 25 38 62
- Use knowledge that subtraction is inverse of addition ( $_{-}23=25$ ,  $23+25=48$ )
- Record mental subtraction using  $-$  and  $=$  signs
- Understand & use related **vocabulary**: take away, subtract, how many are left, how much less is..than..., difference between, how much more is..than..., how many more to make, **decrease**, **inverse**

### Progressions in calculations

$45 - 23$

$15 - 7$

$40 - 19$

$45 - 27$

$120 - 30$

$146 - 50$

$800 - 500$

$819 - 200$

$1200 - 500$

$900 - 7$

$4000 - 3$

$905 - 7$

$4641 - 3$

$372 - 368$  (small difference)

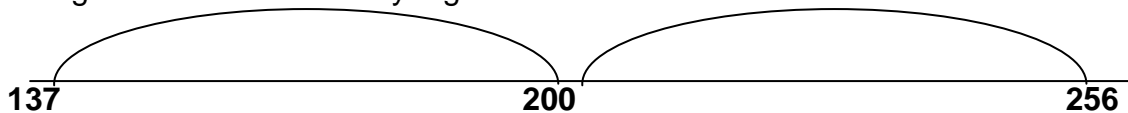
$7003 - 6899$

### Strategies

- Finding a small difference by counting on
- Subtract nearest multiple of ten and adjust
- Use number facts and place value
- Partitioning
- Counting on and back

### Models and Images (As for Year 3)

Using number line efficiently e.g.  $256 - 137$



### Discussing strategies and identifying efficient strategies e.g.

Counting back

$754 - 86$



Counting on

$754 - 86$



If you feel confident that children are ready, move on to expanded method in Stage 5.

**We recommend children should not be using the vertical/column method at this stage.**

**Children should be encouraged to check calculations using the inverse.**

## Stage 5

### Related objectives: Facts, Place Value and Understanding (Mental /Oral Starter)

- Partition 3 digit numbers into multiple of 100, 10 and 1
- Say the number that is 1, 10 or 100 more or less than any given 2 or 3 digit number
- Derive quickly all pairs of numbers that total 100 e.g. 60 40 75 25 38 62
- Count forwards and backwards in steps of 0.1, 0.2, 0.3...
- Derive quickly decimal subtraction facts to 1 e.g. 1 – 0.6
- Use knowledge that subtraction is inverse of addition
- Record mental subtraction using – and = signs
- Understand & use related **vocabulary**: take away, subtract, how many are left, how much less is..than...,difference between, how much more is..than..., how many more to make, **decrease, inverse**

### Progression in mental calculations

45 – 23	4000 – 3
15-7	905 –7
40 – 19	4641 – 3
45 – 27	372 – 368 (small difference)
120 – 30	7003 – 6899
146 – 50	5.7 – 2.5 (decimals)
800 – 500	6.2 – 3.8
819 – 200	0.63 – 0.48
1200 – 500	0.7 – 0.26
900 – 7	

### Strategies (Use Models and Images as for Year 3)

- Partitioning
- Counting on and back (particularly for decimals)
- Find a small difference
- Use number facts and place value

### Written Method

Use the expanded form of decomposition **as preparation for** the compact method where children are secure. TU-TU, to explain the method then move onto HTU – HTU, ThHTU – ThHTU, and decimals **if appropriate** (some pupils may benefit from continuing to use an empty number line for decimals).

e.g. **754 – 486 =**

$$\begin{array}{r} 700 \quad 50 \quad 4 \\ - 400 \quad 80 \quad 6 \\ \hline 300 \quad 70 \quad 8 \end{array} \qquad \begin{array}{r} 700 \quad 40 \quad 14 \\ - 400 \quad 80 \quad 6 \\ \hline 300 \quad 60 \quad 8 \end{array} = 268$$

Extending to the formal contracted form of decomposition:

$$\begin{array}{r} 754 \\ - 486 \\ \hline 268 \end{array}$$

## Stage 6

### Related objectives: Facts, Place Value and Understanding (Mental / Oral Starter)

- Finding a small difference
- Decimals
- Partition 3 digit numbers into multiple of 100, 10 and 1
- Say the number that is 1, 10 or 100 more or less than any given 2 or 3 digit number
- Derive quickly all pairs of numbers that total 100 eg, 60 40 75 25 38 62
- Count forwards and backwards in steps of 0.1, 0.2, 0.3...and 0.25
- Derive quickly decimal subtraction facts to 1 eg  $1 - 0.6$  and  $1 - 0.75$
- Understand & use related **vocabulary**: take away, subtract, how many are left, how much less is..than...,difference between, how much more is..than..., how many more to make, **decrease, inverse**

### Progression in calculations

$45 - 23$

$15 - 7$

$40 - 19$

$45 - 27$

$120 - 30$

$146 - 50$

$800 - 500$

$819 - 200$

$1200 - 500$

$900 - 7$

$4000 - 3$

$905 - 7$

$4641 - 3$

$372 - 368$  (small difference)

$7003 - 6899$

$5.7 - 2.5$  (decimals)

$6.2 - 3.8$

$0.63 - 0.48$

$0.7 - 0.26$

### Strategies (Use models and Images as for Year 3)

- Partitioning
- Counting on and back (particularly for decimals)
- Finding a small difference

### Written Method

Formal contracted form of decomposition (as in Stage 5) using larger numbers and decimals.

$$\begin{array}{r} 9761 \\ - 7593 \\ \hline 2168 \end{array}$$

$$\begin{array}{r} 76.42 \\ - 53.98 \\ \hline 22.44 \end{array}$$

# Multiplication

## Stage 2

### Related Objectives: Facts and Understanding (Mental/Oral Starters)

- Count in 2s and 10s, including 2p and 10p coins
- Count in steps of 3 or 5 using a number track or 100 square
- Know by heart multiplication facts for 2x, x5 and 10x tables
- Know doubles of numbers to at least 15
- Know doubles of multiples of 5 (up to 50)
- To recognise and use x and = signs
- Understand and use related **vocabulary**: double, times, multiply, multiplied by, multiple of, lots of, groups of, times as (big, long, wide)

### Progression in Calculations

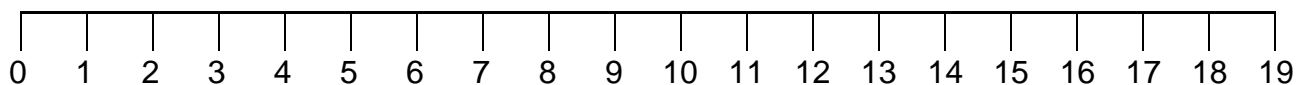
- Multiply a single digit by 1 or 10 (e.g.  $6 \times 10$ ;  $5 \times 1$ )
- Multiply a single digit (1-5) by 2, 3, 4 or 5 (e.g.  $4 \times 3$ )

### Strategies, Models and Images (ITP Multiplication Facts)

Children are introduced to multiplication as:

1. Repeated addition using numbered laminated number lines, drawing on the jumps:

e.g.  $3 \times 2 =$



2. Using an array

\* \*  $3 \times 2 = 6$   
\* \* 3 lots of 2  
\* \* 2 multiplied by 3

$2 \times 3 = 6$

2 lots of 3

3 multiplied by 2

3. Using fingers 2, 4, 6 or 3,6



## Stage 3

### Related Objectives and Strategies (Mental/Oral Starters)

#### Facts and Place Value

- Know by heart multiplication facts for 2x, x3, x4, 5x, x6 and 10x tables
- Know doubles of: whole numbers to at least 20
- Know doubles of multiples of 5 (up to 100)
- Know doubles of multiples of 50
- Observe and describe the effect of multiplying by 1, 10 and 100 using an OHP calculator and place value grid. Develop patterns.

\*1 2 3 4  
10 20 30 40  
100 200 300 400

#### Understanding

- Know that multiplication and division are inverse ( $3 \times 2 = 6$   $6 \div 3 = 2$ )
- Know that halving is the inverse of doubling
- Know that multiplication can be done in any order
- Recognise the x and = sign
- Understand and use related **vocabulary**: double, times, multiply, multiplied by, multiple of, lots of, groups of, times as (big, long, wide)

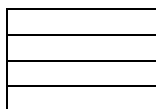
#### Progression in calculation

- Multiply a single digit by 2, 3, 4, 5, 6 or 10
- Multiply a multiple of 10 by 2, 3, 4, 5, 6 or 10 eg.  $20 \times 3$ ,  $40 \times 5$ \*
- Derive division facts from known facts demonstrating an understanding of multiplication e.g.  $9 \times 6 = 10 \times 6 - 6$ ;  $9 \times 7 = 10 \times 7 - 7$ ;  $4 \times 8 = 2 \times 8 + 2 \times 8$ ;  $8 \times 5 = \frac{1}{2} (8 \times 10)$  \*\*
- Multiply a teens number by 2, 3, 4 or 5 e.g.  $13 \times 2$   $15 \times 3$  without crossing the tens boundary

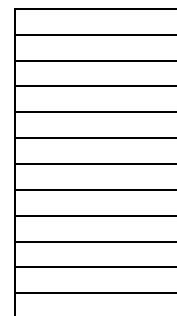
#### Strategies, Models and Images (See Year 2 for related vocabulary)

1. Using an array (see Stage 2)
2. Repeated addition: e.g.  $6 \times 3 = 6 + 6 + 6 = 3 \times 6 = 3 + 3 + 3 + 3 + 3 + 3$

3. Scaling e.g. Make higher.



a tower of 4 cubes, 3



times

4. Pupils should calculate up to  $6 \times$  mentally. Scaffold this either by using fingers, recording multiples e.g. 6, 12, 18 or blank number lines e.g.  $6 \times 3$



3. Using counting stick/empty number line to derive new facts from known facts\*\*

4. Partitioning tens and ones start with teens numbers e.g.  $12 \times 3 = 10 \times 3 + 2 \times 3 = 30 + 6 = 36$

5. Partitioning 2 digit numbers e.g.  $35 \times 6 = 30 \times 6 + 5 \times 6$



## Stage 4

### Related objectives (Mental/oral starters)

#### Facts

- Know by heart multiplication facts up to 10 x10 and derive division facts
- Know doubles of numbers to 50
- Know doubles of multiples of 10 (up to 500)
- Know doubles of multiples of 100 (up to) to 5000

#### Understanding

- Know that multiplication and division are inverse ( $6 \times \_ = 30$   $30 \div 6 = 5$ )
- Know that halving is the inverse of doubling
- Know that multiplication can be done in any order
- Understand and use related **vocabulary**: multiply, multiplied by, multiple, product, inverse

### Progression in mental calculations

- Multiply a single digit by single digit
- Multiply a multiple of 10 by U e.g.  $20 \times 3$ ,  $40 \times 5$
- Multiply a 2 digit number by U without crossing the tens boundary eg  $23 \times 2$   $32 \times 3$
- Multiply 2 digit number by U crossing the tens boundary e.g.  $13 \times 5$

### Strategies

- Using any known facts to derive new facts
- Multiply by 10 and 100 by shifting digits
- Multiply by 4 (double and double again)
- Multiply by 5 (times by 10 and then halve)
- Multiply by 20 ( times by 10 and then double)
- Multiply by 9 or 11 (times by 10 and adjust)

### Mental Method with recording

Children should be introduced to a mental method, with jottings, using partitioning:

TU x U

$$12 \times 3 = (10 \times 3) + (2 \times 3) = 30 + 6 = 96 \text{ OR}$$

$$\mathbf{12 \times 3 = 30 + 6 = 96 \text{ (more efficient recording)}}$$

$$38 \times 7 = (30 \times 7) + (8 \times 7) = 210 + 56 = 266 \text{ OR}$$

$$\mathbf{38 \times 7 = 210 + 56 \text{ (more efficient recording)}}$$

If children are secure in their mental calculations, strategies and partitioning **then** introduce the grid layout (*Use Grid Method ITP*)

Written Method:

x	30	8	
7	210	56	266

## Stage 5

### Related objectives (Mental / Oral Starter)

#### **Facts**

- Know by heart multiplication facts up to 10 x10 and derive division facts
- Doubles of multiples of whole numbers to 100
- Doubles of multiples of 10 to 1000
- Doubles of multiples of 100 to 10000
- Know square numbers

#### **Understanding**

- Know that multiplication and division are inverse ( $30 \times \_ = 300$ ,  $300 \div 30 = 10$ )
- Know that halving is the inverse of doubling
- Know that multiplication can be done in any order
- Understand and use related **vocabulary**: multiply, multiplied by, multiple, product, inverse

### Progression in calculations

- Multiply a single digit by single digit
- Multiply a multiple of 10 by U eg.  $20 \times 3$ ,  $40 \times 5$
- Multiply a 2 digit number by U without crossing the tens boundary eg  $23 \times 2$   $32 \times 3$
- Multiply 2 digit number by U crossing the tens boundary e.g.  $13 \times 5$
- Multiply a 2 then 3 digit number by multiples of 10 and 100 e.g.  $23 \times 600$
- Multiply a 3 digit number by U
- Multiply a 3 digit number by TU

#### **Strategies**

- Multiply by 10 or 100 by shifting digits
- Use factors eg.  $8 \times 12 = 8 \times 4 \times 3$
- Multiply by 25 (times by 100 and divide by 4)
- Multiply by 4 (double and double again)
- Multiply by 5 (times by 10 and then halve)
- Multiply by 20 (times by 10 and then double)
- Double 2 digit numbers by partitioning e.g. double 23 = double 20 and double 3

#### **Written Method**

Extend multiplication to decimals with one place

HTU x U, leading to TU x TU, using grid layout only:

e.g.  $56 \times 27 =$

x	50	6	
20	1000	120	1120
7	350	42	392
			1512

## Stage 6

### Related objectives

#### Facts

- Know by heart multiplication facts up to  $12 \times 12$
- Doubles of multiples of whole numbers to 100
- Doubles of 2 digit numbers (e.g.  $3.8 \times 2$ )
- Doubles of multiples of 10 to 1000
- Doubles of multiples of 100 to 10000
- Derive square numbers to  $12 \times 12$
- Squares of multiples of 10 e.g.  $60 \times 60$  (up to 100)

#### Understanding

- Know that multiplication and division are inverse ( $36 \times \_ = 3600$ ,  $3600 \div 36 = 100$ )
- Know that halving is the inverse of doubling
- Know that multiplication can be done in any order
- Understand and use related **vocabulary**: times, multiply, multiplied by, multiple, product, inverse

### Progression in mental calculation

- Multiply a single digit (1-5) by 2, 3, 4 or 5
- Multiply a single digit by single digit
- Multiply a multiple of 10 by 2, 3, 4, 5 or 10 eg.  $20 \times 3$ ,  $40 \times 5$
- Multiply a 2 digit number by 2, 3, 4 or 5 without crossing the tens boundary eg  $23 \times 2$   $32 \times 3$
- Multiply 2 digit number by 2, 3, 4, or 5 crossing the tens boundary e.g.  $13 \times 5$
- Multiply a 2 digit number by 6,7,8 or 9
- Multiply a 2 then 3 digit number by multiples of 10 and 100 e.g.  $23 \times 600$
- Multiply a decimal number by a single digit

#### Strategies

- Multiply by 10 or 100 by shifting digits
- Use factors e.g.  $8 \times 12 = 8 \times 4 \times 3$
- Multiply by 25 (times by 100 and divide by 4)
- Multiply by 4 (double and double again)
- Multiply by 5 (times by 10 and then halve)
- Multiply by 20 ( times by 10 and then double)
- Multiply by 49 and 51 (times by 50 and adjust)

### **Written Method**

Children using standard compact method

$$\begin{array}{r} 275 \\ \times \quad 8 \\ \hline 2200 \\ \quad 64 \\ \hline \end{array}$$

# Division

## Stage 2

### Related Objectives: Facts and Understanding (Mental / Oral Starters)

- Find a difference by counting on or back, bridging through 10 and 100
- To use number facts and place value to subtract mentally;
- Count forwards and backwards in 2s and 10s, including 2p and 10p coins
- Count forwards and backwards in steps of 3 or 5 using a number track or 100 square
- To know multiplication and division facts for 2 and 10 times.
- Halve any multiple of 10 (up to 100)
- Divide any 2 digit multiple of 10 by 1 or 10 e.g.  $60 \div 10 = 6$
- To recognise and use  $\div$  and  $=$  signs
- Understand and use related **vocabulary**: each, share, halve, divide, left over, divided by, equal groups of

### Progression in calculation

- Divide single digit multiples of 2,3,4, and 5 by 2,3,4 or 5 e.g.  $6 \div 2$
- Divide 2 digit multiples of 2, 3, 4, and 5 by 2, 3, 4 or 5 e.g.  $24 \div 3$
- Divide a 2 digit multiple of 10 by 1 or 10 e.g.  $40 \div 10$

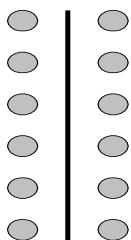
### Strategies, Models and Images

Children need to understand division as **sharing** and then as **grouping** (ITP: *Groupn*)

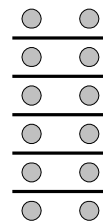
Demonstrate practically (e.g. using OHP counters):

$$12 \div 2$$

sharing (divided into/between)



grouping (divided by)

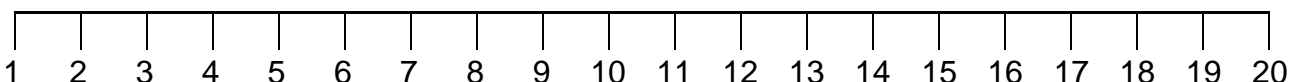


**Sharing:** 20 cakes shared between 4 people - give one cake to each person and keep going until all cakes are used. How many cakes does each person have?

**Grouping:** 24 eggs are packed in boxes of 6. How many boxes are needed? Take 6 eggs and pack the first box, and continue until there are no eggs left. Count how many boxes were needed.

Use of laminated number track for repeated subtraction (grouping):

e.g.  $15 \div 3 =$



## Stage 3

### Related Objectives (Mental / Oral Starters)

- Find a difference by counting on;
- To use number facts and place value to subtract mentally;

### **Facts**

- Know by heart multiplication and division facts for 2x, x3, x4 5x, x6 and 10x table
- Count forwards and backwards in steps of 3 and 4
- Know halves of even numbers to at least 20
- Know halves of multiples of 10 (up to 100)
- Know halves of multiples 100 up to 1000

### **Understanding**

- Know that multiplication and division are inverse ( $\_ \div 3 = 2$ ,  $2 \times 3 = 6$ )
- Know that halving is the inverse of doubling
- Recognise the  $\div$  and  $=$  sign
- Observe and describe the effect of multiplying and dividing by 1, 10 and 100 using an OHP calculator and place value grid. Develop patterns.

1 2 3 4  
10 20 30 40  
100 200 300 400

- Understand and use related **vocabulary**: each, share, halve, divide, left over, divided by, equal groups of, remainder

### Progression in calculation

- Divide 2 digit multiples of 2, 3, 4, and 5 by 2, 3, 4 or 5
- Divide a 2 digit multiple of 10 by 1 or 10
- Divide any 3 digit multiple of 100 by 10 or 100
- Find remainders, initially in **practical** contexts

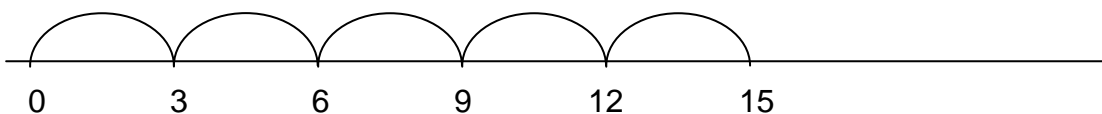
### Strategies, Models and Images

As for Stage 2, using OHP counters and number tracks, then extend to children using blank number lines for repeated subtraction.

Use Interactive Teaching Programme: Grouping, to demonstrate,

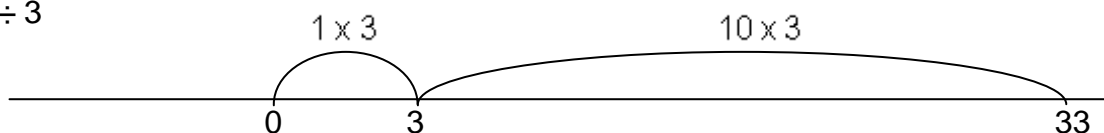
e.g.  $15 \div 3$

Counting groups forwards and backwards



Children could use number line, fingers or an array.

$33 \div 3$



Missing out demonstrating remainders?

# Stage 4

## Related objectives(Mental/oral starters)

### Facts

- Know by heart multiplication and division facts for all facts up to  $10 \times 10$
- Count forwards and backwards in steps of 6 and 8
- Know halves of even numbers to at least 100
- Know halves of multiples of 10 (up to 500)
- Know halves of multiples of 100 (up to 5000)

### Understanding

- Know that multiplication and division are inverse ( $\_ \div 3 = 4$ ,  $3 \times 4 = 12$ )
- Know that halving is the inverse of doubling
- Recognise the  $\div$  and  $=$  sign
- Understand and use related **vocabulary**: share, halve, divided by, divisible by, divided into, factor, quotient, remainder, inverse

## Progression in calculation

- Divide 2 digit multiples of 2, 3, 4, and 5 by 2, 3, 4 or 5
- Divide a 2 digit multiple of 10 by 1 or 10
- Divide any 3 digit multiple of 100 by 10 or 100
- Give a remainder as a whole number
- Decide whether to round up or round down after division in word problem context:

Rounding up:        *I have 62 cakes. One box holds 8 cakes.  
How many boxes do I need to hold all the cakes?*

Rounding down:    *I have £62. Tickets cost £8. How many people can I take?*

## Strategies

- Find quarters by halving halves
  
- Find eighths by halving quarters
- Divide by 10 and 100 by shifting digits
- Partition to find multiples of the divisor (chunking)  $66 \div 5 = 50 \div 5$  and  $15 \div 5$
- Find half by partitioning (place value) e.g. half 56 = half 50 and half 6

## Models and Images

Using jottings and notes to support and informal recording:

**Pupils need lots of opportunities to partition by multiples of the divisor.** Encourage pupils to use knowledge of multiples of 10 of the divisor.

$$84 \div 7$$

$$70 + 14$$

$$10 \quad 2$$

$$12$$

# Stage 5

## Related objectives (Mental/oral starters)

### Facts

- Know by heart multiplication and division facts for all tables to 10 x 10
- Count forwards and backwards in steps of 7 and 9
- Know halves of even numbers to at least 100
- Know halves of multiples of 10 (up to 1000)
- Know halves of multiples of 100 (up to 10000)

### Understanding

- Know that multiplication and division are inverse ( $\_ \div 3 = 10$ ,  $3 \times 10 = 30$ )
- Know that halving is the inverse of doubling
- Recognise the  $\div$  and  $=$  sign
- Understand and use related vocabulary: share, halve, divided by, divisible by, divided into, factor, quotient, remainder, inverse

## Progression in calculations

- Divide 2 digit multiples of 2, 3, 4, and 5 by 2, 3, 4 or 5
- Divide any 2 digit number by any single digit number
- Divide a 3 digit number by a single digit number
- Divide a 3 digit number by a 2 digit number
- Divide a 3 digit multiple of 10 by 1 or 10
- Divide any 4 digit multiple of 100 by 10 or 100
- Decide whether to round up or round down after division in word problem
- Interpret the calculator display
- To begin to give the quotient (answer) as a decimal and a fraction

$$43 \div 4 = 10 \frac{3}{4}$$

$$61 \div 4 = 15.25$$

$$£5.40 \div 4 = £1.35$$

### Strategies

- Find quarters by halving halves
- Find eighths by halving quarters
- Find sixths by halving thirds
- Divide by 10 and 100 by shifting digits
- Partition to find multiples of the divisor (chunking)  $66 \div 5 = 50 \div 5$  and  $15 \div 5$
- Find half by partitioning (place value) e.g. half 156 = half 100, half 50 and half 6
- Use factors e.g.  $90 \div 6 = 15$  using  $3 \times 2 = 6$      $90 \div 3 = 30$      $30 \div 2 = 15$

### Written Method

Children can continue to use informal method from Stage 4 both when dividing by U and TU. Children can also continue to record using known facts. Alternatively pupils can record it more formally using the chunking method.

$$222 \div 3 =$$

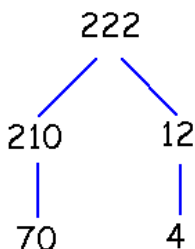
I know  $3 \times 7 = 21$   $\therefore$

$$3 \times 70 = 210$$

$$222 - 210 = 12$$

I know that  $4 \times 3 = 12$

$$70 + 4 = 74$$



OR

$$\begin{array}{r} 222 \\ - 210 \quad (70 \times 3) \\ \hline 12 \\ - 12 \quad (4 \times 3) \\ \hline 0 \end{array}$$

## Stage 6

### Related objectives (Mental/oral starters)

#### Facts

- Know by heart multiplication and division facts for 2x, 3x, 4x, 5x, 6x, **7x**, 8x, **9x** and 10x table
- Know halves of even numbers to at least 100
- Know halves of multiples of 10 (up to 1000)
- Know halves of multiples of 100 (up to 10000)
- Know square root of numbers to 100

#### Understanding

- Know that multiplication and division are inverse ( $\_ \div 3 = 36$ ,  $36 \times 3 = 108$ )
- Know that halving is the inverse of doubling
- Recognise the  $\div$  and  $=$  sign
- Understand and use related **vocabulary**: share, halve, divided by, divisible by, divided into, factor, quotient, remainder, inverse

### Progression in calculations

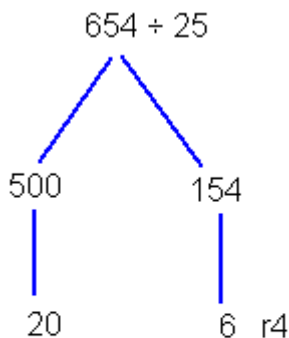
- Divide 2 digit multiples of 2, 3, 4, and 5 by 2, 3, 4 or 5
- Divide any 2 digit number by any single digit number
- Divide a 3 digit number by a single digit number
- Divide a 3 digit number by a 2 digit number
- Divide a 3 digit multiple of 10 by 1 or 10
- Divide any 4 digit multiple of 100 by 10 or 100
- Decide whether to round up or round down after division in word problem
- Interpret the calculator display
- To begin to give the quotient (answer) as a decimal and a fraction  
 $43 \div 4 = 10 \frac{3}{4}$   
 $61 \div 4 = 15.25$   
 $\pounds 5.40 \div 4 = \pounds 1.35$
- Use decimals to up 2 places

#### Strategies

- Find quarters by halving halves
- Find eighths by halving quarters
- Find sixths by halving thirds
- Divide by 10 and 100 by shifting digits
- Partition to find multiples of the divisor (chunking)  $66 \div 5 = 50 \div 5$  and  $15 \div 5$
- Find half by partitioning (place value) e.g. half 156 = half 100, half 50 and half 6
- Use factors e.g.  $378 \div 21 = 18$  using  $3 \times 7 = 21$   $378 \div 3 = 126$   $126 \div 7 = 18$

#### Written Method

Pupils can continue to use methods from Stage 5 and when ready move on to the 'bus stop' method. However it remains more efficient to use chunking or informal jottings when  $\div$  TU.



OR

$$\begin{array}{r} 654 \\ - 500 \quad (20) \\ \hline 154 \\ - 150 \quad (6) \\ \hline 4 \end{array}$$

$$20 + 6 = 26$$

remainder 4

'Bus stop' method

$$325 \div 7 =$$

$$\begin{array}{r} 46 \\ 7 \overline{) 325} \\ \underline{28} \phantom{0} \\ 45 \\ \underline{42} \\ 30 \\ \underline{28} \\ 20 \\ \underline{14} \\ 6 \end{array}$$